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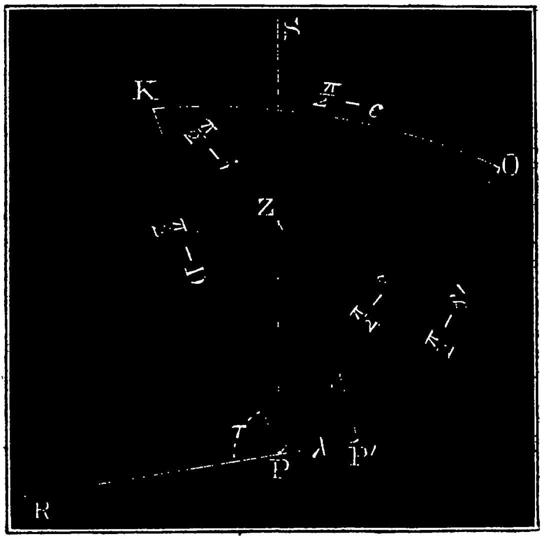
To adjust the Polar Axis of an Equatorial Telescope for Photographic Purposes. By Professor Arthur A. Rambaut, D.Sc.

An equatorial telescope intended for ordinary visual work may be considered to be adjusted with sufficient accuracy, as far as its polar axis is concerned, if the errors in altitude and azimuth do not exceed a single minute of arc. In most cases, too, the ordinary method of adjusting the axis by readings of the circles is not susceptible of higher accuracy unless elaborate precautions are taken to eliminate the effect of flexure.

When, however, a telescope is intended for photographic purposes it is advisable to orient the axis with a much higher degree of precision than this. In the *Bulletin du Comité International Permanent pour l'exécution photographique de la Carte du Ciel*, Tome 1^er^, p. 385, Dr. Scheiner points out a method by which this may be accomplished. It has, however, the disadvantages that the corrections have to be made tentatively during the progress of the observations, and are affected by the variations in the refraction as the star changes its position.

In the method which I propose the errors in the position of the axis are determined by measures of a photograph on which two short exposures of a star have been taken at an interval of 15 or 30 minutes, the driving-clock of the instrument being allowed to run without hand correction in the meanwhile.

If the axis is not perfectly adjusted the two images will not fall on exactly the same point of the plate, and measures of the distance between the two images when corrected for the effect of refraction will afford the material for determining the position of the axis.



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I assume, for the present, that the driving-clock is running accurately to sidereal time, and that the errors of the instrument have been reduced to within a few minutes of arc.

Let P be the pole of the heavens, and PS the meridian towards the south at S; Z the zenith; P′ the pole of the instrument; K the point in which the extremity of the declination axis if produced would meet the sphere; and O the point towards which the telescope is direced.

Let the arc PP′ be denoted by λ,
,, ,, KP′ ,, $\frac{\pi}{2}-i$,
,, ,, KO ,, $\frac{\pi}{2}-c$,
,, ,, KP ,, $\frac{\pi}{2}-\mathrm{D}$,
,, ,, ZP′ ,, ζ,
,, angle SPP′ ,, h,
,, ,, SPO ,, θ,
,, ,, KPR ,, τ.

Then K describes a circle round P′ and of course PP′ is fixed. Let t_0 be the reading of the hour-circle when the point K is on the great circle PP′, and t the reading when the telescope is directed to a point O west of the meridian. Then the angle KP′P is equal to $t-t_0$. We thus obtain from the triangle KPP′

$$\sin \mathrm{D} = \sin i \cos \lambda + \cos i \sin \lambda \cos (t - t_\theta),$$

and

$$\sin \tau \cos \mathrm{D} = \cos i \sin (t - t_0),$$

or to the first order of small quantities,

$$\left.\begin{aligned} \mathrm{D} &= i + \lambda \cos (t - t_0), \\ \text{and} \quad \tau &= t - t_0. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We see also that θ is the hour-angle of the telescope. Let us put δ equal to the true, and δ' equal to the instrumental, declination of the point O; hence δ' is constant throughout the exposures.

If now the object-glass end of the telescope makes an angle $90° - c$ with the declination axis, then $\mathrm{KO} = 90° - c$, and we have

$$\sin c = \sin \mathrm{D} \sin \delta + \cos \mathrm{D} \cos \delta \cos (\varpi + \theta - \tau - h),$$

or

$$c = \mathrm{D} \sin \delta + \cos \delta \sin \left(\tau + h - \theta - \frac{\pi}{2}\right).$$

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Hence unless the star under observation is within a few minutes of the pole

$$\sin\left(\tau + h - \theta - \frac{\pi}{2}\right)$$

is a small quantity, for which we may write simply

$$\tau + h - \theta - \frac{\pi}{2}.$$

Thus we obtain

$$\theta = \tau + h - \frac{\pi}{2} - c \sec \delta + \mathrm{D} \tan \delta;$$

or, substituting from (1), we find

$$\theta - t = -t_0 + h - \frac{\pi}{2} - c \sec \delta + i \tan \delta + \lambda \tan \delta \cos (t - t_0) \quad . \; . \; . \quad (2)$$

whence

$$d(\theta - t) = -\lambda \tan \delta \sin (t - t_0) dt - [c \sec \delta \tan \delta + i \sec^2 \delta + \lambda \sec^2 \delta \cos (t - t_0)]\, d\delta.$$

Again, we have from the triangle PP′O

$$\sin \delta' = \sin \delta \cos \lambda + \cos \delta \sin \lambda \cos (\theta - h)$$
$$= \sin \delta + \lambda \cos \delta \cos (\theta - h);$$

but from equation (2) we have, to a first approximation,

$$\frac{\pi}{2} + \theta - h = t - t_0,$$

and therefore

$$\sin \delta' - \sin \delta = \lambda \cos \delta \sin (t - t_0),$$

from which we find, to the same degree of approximation,

$$\delta = \delta' - \lambda \sin (t - t_0),$$

and consequently

$$d\delta = -\lambda \cos (t - t_0)\, dt.$$

We thus obtain

$$\left.\begin{aligned} d(\theta - t) &= -\lambda \tan \delta \sin (t - t_0)\, dt, \\ \text{and} \quad d\delta &= -\lambda \cos (t - t_0)\, dt. \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

Again, if ϕ denotes the latitude, we have

$$\cos \zeta = \sin \phi \cos \lambda + \cos \phi \sin \lambda \cos h = \sin \phi + \lambda \cos h \,.\, \cos \phi.$$

Let $\zeta + k = 90° - \phi$, *i.e.* let the pole of the instrument be *too high* by the angle k; then

$$\cos \zeta = \sin (\phi + k) = \sin \phi + k \cos \phi,$$

and

$$\sin \zeta = \cos \phi - k \sin \phi.$$

Hence

$$k = \lambda \cos h.$$

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If also we take A to be the azimuth of the point P′ measured from the *north* towards *west* we have

$$\sin A = \frac{\sin\lambda \sin h}{\sin\zeta} \text{ or } A = \lambda \sin h \,.\, \sec\phi.$$

Now, if the telescope is in the first instance pointed to a star, then $d(\theta - t)$ denotes the difference in hour-angle between the star and the telescope at the end of the interval dt, or $d(\theta - t)\cos\delta$ represents the displacement of the star's image on the plate in the direction of the parallel. Also $d\delta$ is the displacement at right angles to the parallel. Hence if we denote these displacements by X and Y respectively, we have the following equations:

$$\left.\begin{aligned} X &= d(\theta - t)\cos\delta = -\lambda \sin\delta \cos(\theta - h)\frac{15dt}{R}, \\ Y &= d\delta \qquad\qquad\;\; = \lambda \sin(\theta - h)\frac{15dt}{R} \\ &(\text{where } R = 206265), \\ k &= \lambda \cos h, \\ \text{and} \quad A &= \lambda \sin h \,.\, \sec\phi. \end{aligned}\right\} \quad . \; . \; . \quad (4)$$

From these we obtain

$$\left.\begin{aligned} k &= -\left\{\frac{X}{\sin\delta}\cos\theta - Y\sin\theta\right\}\frac{R}{15dt}, \\ \text{and} \quad A &= -\left\{\frac{X}{\sin\delta}\sin\theta + Y\cos\theta\right\}\frac{R}{15dt} \,.\, \sec\phi, \end{aligned}\right\} \quad . \; . \; . \; . \quad (5)$$

in which θ is the hour-angle of the star at the middle of the interval between the exposures.

With regard to the signs of the quantities k, A, X, and Y we see

(α) That k is the angle through which the pole of the instrument must be *lowered*.

(β) That A is the angle through which the *northern* end of the mounting must be turned towards the *east*.

(γ) From the first of equations (4) we find that X is positive when $d\theta > dt$, i.e. when the instrument moves faster than the star. Hence the second image will fall to the *following* side of the first, or the *following* is the *positive* direction of X.

(δ) From the second of equations (4) we see likewise that Y is positive when $d\delta$ is positive, or when the telescope at the end of the exposure is directed to a more northern point than at the beginning; consequently the second image will be *south* of the first.

Hence we see that taking as axes of X and Y, the projections of the parallel and meridian through the first image, and the plate being placed so that the stars on it appear as seen in an inverting telescope, the positive and negative directions of the

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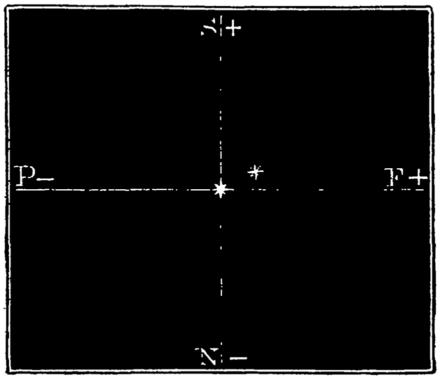
axes will be disposed in the usual manner as shown in the diagram.



The Refraction :—If the star whose image we are considering is not very close to the pole, the effect of the change in the refraction during the exposure must be taken into account when the highest accuracy is desired in the adjustment of the instrument.

This correction may be applied very conveniently by means of the formulæ which I published in the *Astronomische Nachrichten*, No. 3125. In the case before us we have $d\delta$ equal to zero, and da equal to dt; and since we need only consider the terms of the first order the formulæ become

$$\Delta X' = \beta \,.\, \frac{\cos\phi\,(\cos\delta\cos\phi + \sin\delta\sin\phi\cos\theta)}{\{\sin\delta\sin\phi + \cos\delta\cos\phi\cos\theta\}^2}\,\frac{15\,dt}{R}$$

and
$$\Delta Y' = \beta \,.\, \frac{\cos\phi\sin\phi\sin\theta}{\{\sin\delta\sin\phi + \cos\delta\cos\phi\cos\theta\}^2}\,\frac{15\,dt}{R}.$$

If, then, X′ and Y′ are the measured displacements along and at right angles to the parallel, we have

$$X = X' + \Delta X' \qquad \text{and } Y = Y' + \Delta Y'.$$

Having thus determined k and A by means of equations (5), the corrections may very easily be effected in the following manner:—

(1) *To adjust in altitude*, point the telescope to a star near the meridian and not very far from the pole. Set the star on the horizontal wire, and by means of the scale ordinarily found at the eye-end of the pilot or guiding telescope, displace the wire in declination through an angle equal to k. Using the adjusting screws at the base, bring the wire once more into coincidence with the star. The axis will then be correctly adjusted in altitude.

(2) *To adjust in azimuth*, point the telescope on a star near the meridian and not far from the equator. Having set the star on the vertical wire, displace the latter through an angle D [$=\text{A}\sin(\phi-\delta)$]. When the wire is brought once more into coincidence with the star by means of the adjusting screws at the

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base of the instrument, the axis will be correctly adjusted in azimuth.

Irregularities in the Clock.—In what precedes I have supposed that the clock was running correctly to sidereal time.

In this connection there are two cases to be considered. If the clock is one of Sir Howard Grubb's latest pattern, in which the rate is electrically controlled by a sidereal pendulum clock, this condition will be fulfilled.

If, however, the clock is one whose rate may be depended upon to remain constant during the exposure, but without control from a pendulum clock, it may be rated to keep sidereal time by observation of a star at the equator. For the first of equations (3) shows that $d\,(\theta-t)$ is zero for such a star. Hence, in order to rate the clock correctly, it is only necessary to direct the telescope to a star on the equator and to adjust its rate until it keeps pace with the star.

But if the rate cannot be depended on to remain constant during the interval between the exposures, since the principle of the method will not allow of any hand correction being introduced, it will be necessary to reject the displacement in R.A. and to depend entirely on that in declination. We can, however, by properly selecting the stars, obtain both k and A from these displacements. For if in the first instance we take a star on the meridian, since θ is then equal to zero, we find from equation (5) that

$$A=-\frac{YR}{15\,dt}\sec\phi;$$

and if we next take a star at a distance of 6^h from the meridian, we have

$$k=\frac{YR}{15\,dt}.$$

The Observatory, Dunsink, Co. Dublin:
1893 *December* 1.

Note on a Star-correction Facilitator, or an instrument for readily obtaining the products of pairs of numbers, one number being taken from one set (e.g. *day-constants*) *and the second from another set* (e.g. *star-constants*).

By T. C. Hudson, B.A. (Cantab.)

(*Communicated by A. M. W. Downing.*)

This instrument, which may for the present be briefly described as an antilog.-roller with a set of four shiftable windowed sheaths, was primarily devised for the sake of minimising the labour involved in the calculation of star ephemerides. The principle may be explained as follows:—